Using Yield Curve Shapes to Manage Bond Portfolios

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Multiple discriminant analysis can be used to extract and utilize the information contained in the shape of the yield curve. Using this information, combined with economic variables such as the unemployment and inflation rates, the MDA model classified months in the 1966–87 period as either "bear" or "bull" months. A portfolio based on the model's predictions turned in a performance superior to that of a buy-and-hold strategy. This suggests that MDA can enhance the returns of investment portfolios.

This article describes a statistical model that enables a portfolio manager to anticipate changes in the yield levels of government bonds by analyzing yield curve shapes and selected economic data. The statistical technique underlying the model is stepwise multiple discriminant analysis (MDA).[1] MDA calculates optimal weightings for variables that describe a group of objects and then assigns each object to one of two categories. In this study, the objects are 12-month changes in the interest rate levels of 10-year Treasury bonds during the 1966–87 period. The two categories to which we assign these object are "bull market" (lower 10-year interest rates) and "bear market" (higher 10-year interest rates).

The mathematical form of the basic MDA prediction equation used to assign each month in this study to its appropriate category is:

$$Z_{category} = k_1x_1 + k_2x_2 + k_3x_3 + k_4x_4 + k_5$$

The x's represent four different data variables. x_1 is the yield curve slope (10-year yield minus three-year yield divided by the 10-year yield). This is a more efficient statistic than the difference itself, because it tends to avoid the bias introduced by any systematic drift in the absolute level of interest rates.

x_2 is yield curve torque (having a value of 1 if the current difference between the 10 and the three-year yields is greater than its 36-month moving average and a value of 0 if the current difference is less than its 36-month moving average). This is a measure of whether the yield curve is steep or inverted, as compared with its previous three-year average shape.

x_3 is the monthly percentage change in the Consumer Price Index (CPI) lagged one month to take into account reporting delays.

x_4 is the unemployment rate lagged one month to take into account reporting delays.

The yield curve slope and yield curve torque variables were chosen as the most descriptive measures of the dynamic changes in yield curve shape this study seeks to evaluate. The inflation and unemployment variables were chosen as broad measures of economic condition. Politicians, the media, sociologists and economists most often use these variables in describing the state of the economy.

For both bull and bear categories, the MDA process identifies a set of optimal weightings defined as k's. These weightings are those that produce Z values that are most accurate in assigning each object to the correct category. Using the data from each 12-month period, we calculated a Z value for both categories. The model assigns a period to the category with the largest Z value.

THE MODEL

We collected data for each of the four variables for the 270 monthly periods beginning January 1966. The first 200 months were designated the "original sample" and used to develop the model. The remaining 70 months became the "holdout sample," used later for a more independent test of the model's predictive ability.

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As a first step in the analysis of the original sample, we assigned each month to its correct category—bull or bear. The MDA process then analyzed the original sample's monthly data for each of the four variables and produced the results shown in Table 1.

Table 1. Variable Means and Test of Significance

Variable	Name	Bear Group Mean	Bull Group Mean	F-Stat.
x1	Yield Curve Slope	1.10	2.24	17.6
x2	Yield Curve Torque (0 or 1)	0.39	0.73	20.5
x3	Mo. CPI Change (%)	0.59	0.48	21.6
x4	Unemployment Rate (%)	5.4	6.7	29.1

Compare the average values of each variable within the bull and the bear categories; there is a noticeable difference. Formal tests of these observed differences resulted in high F-values (also shown in Table 1). At the 1% significance level, the F-statistic, $F_{4,195}$, is 3.32. As all of the calculated F-values are considerably larger than 3.32, the average values for each of the four variables differ significantly between the bull and bear categories. The null hypothesis that the bull and bear market averages of each of the four variables are not different (and that it is thereby impossible to anticipate the state of the market one year later) can be rejected at the 1% significance level.

Using the means and distributions of each variable during the original sample period, the MDA process next calculated the classification equation for the bull and bear categories. We found the following equations best able to assign each month to its correct category:

$$Z_{Bear} = -48x_1 + 5.2x_2 + 2.1x_3 + 2.6x_4 - 9.0,$$

$$Z_{Bull} = -71x_1 + 7.8x_2 + 0.1x_3 + 3.5x_4 - 14.5.$$

RESULTS

The predictions of any discriminant model are subject to the errors inherent in any two-choice decision process. In the context of this MDA application, these errors are (1) predicting a bear market when a bull market will actually happen or (2) predicting a bull market when it will actually be a bear market. These errors are often called Type I and Type II, respectively.

To evaluate the model's ability to describe the state of the market one year later, we measured the occurrence of each type of error by testing both the original and holdout samples.

Original Sample

As data during the first 200 months are used to calculate the classification equations, the MDA process during this 200-month period is naturally biased toward accurate classification of the state of the market one year later. To reduce this bias, we employed the jackknife classification technique, wherein each case is classified into the group with the highest posterior probability according to classification equations computed from all the data except data for the case being classified. This is a special case of the general cross-validation method, in which the classification equations are computed on a subset of data and the probability of misclassification is estimated from the remaining data. When each piece of data is left out in turn, the method is known as the "jackknife."

We next calculated values for both classification functions for each of the 200 months in the original sample. The MDA process assigned each of these months to the category whose MDA classification equation from the original sample data had the largest value. Table 2 presents the results of the jackknife classification.

Table 2. Original Sample Classifications

Actual Group	Predicted Group: Bear	Predicted Group: Bull
Bear	117	26
Bull	17	40

The percentage of correct classifications was 82% (117 out of 143) for bear markets and 70% (40 out of 57) for bull markets, resulting in a combined correct classification of 79 out of 100 cases. The better predictive ability for bear markets appears to reflect the more frequent occurrence of bear market months during the time period (143 bear months versus 57 bull months). This imbalance, while not statistically desirable, was unavoidable, as the original sample period occurred mostly during the secular bear market of the 1960s and 1970s.

Holdout Sample

Use of the original sample, although jackknifed, results in significant sampling bias because of the time-series nature of the data. The classification of other and more independent time periods is likely to be considerably less biased.

The holdout sample data chronologically follow the original sample period, hence are likely to gauge the model's predictive ability with less bias. The MDA classification equations produced using the first 200 months of data were applied to the next 70 months. Table 3 gives the results. The number of correct predictions using the original sample data to classify each of the months (numbers 201 through 270) in the holdout sample is 44, for a success percentage of 63%.

Table 3. Holdout Sample Classifications

Actual Group	Predicted Group: Bear	Predicted Group: Bull
Bear	18	18
Bull	8	26

While considerably less than the success rate for the original sample (Table 2), the level of success for the holdout sample period is statistically much greater than chance alone. Specifically, in a binomial experiment with 70 identical and independent trials, the probability of getting 45 or more correct guesses out of 70 trials merely by chance is only 1.1%.[2] These results imply that the MDA process of analyzing the yield curve and selective economic data can be an important source of information for portfolio managers.

SIMULATED PORTFOLIO

The true measure of the model's value to a portfolio manager is its ability not only to detect a bull or a bear market, but also to generate excess return relative to some appropriate benchmark index. To put a monetary value on the accuracy of the holdout sample predictions, we used the following simulated portfolio strategy.

For any month classified as a bull market, we invested the portfolio during the following month in a 10-year, zero-coupon bond with a yield equal to that of a 10-year Treasury bond. For any month classified as a bear market, we invested the portfolio during the following month in a three-year, zero-coupon bond with a yield equal to that of a three-year Treasury bond.

We also calculated returns from three passive strategies, for comparison purposes. The first passive strategy called for continuous investment in the three-year zero bond throughout the 70-month period. The second called for continuous investment in the 10-year zero bond. The third was invested 50% in the three-year and 50% in the 10-year bond. Table 4 presents the annualized rates of return from the three passive and the MDA simulated strategy during the 70-month holdout sample period.

Table 4. Annualized Rates of Return of Passive and Active Strategies, 1982–87

1. Constant Maturity Three-Year	10.30%
2. Constant Maturity 10-Year	13.09%
3. Half-and-Half	11.74%
4. Holdout Sample	13.53%

Note that, while the original sample data primarily covered a bear market environment, the holdout sample represents a period broadly characterized as a bull market. We believe it is significant that a strategy based on the classification equations obtained in a bear market environment could provide material incremental investment return in a subsequent bull market environment. This aspect seemingly adds to the robustness of the predictive abilities of the MDA methodology.

CONCLUSIONS

The analytical process used in this study provides a quantitative way of capitalizing on two yield curve behaviors that have been thoroughly described in the academic literature.[3] (1) Interest rates display mean-reversion behavior; steep yield curves tend to flatten eventually and flat yield curves tend to get steeper. (2) The economy experiences phases of expansion and contraction, which are reflected in the behavior of the yield curve.

We believe that our statistically significant results demonstrate that mathematical predictive techniques for analyzing yield curve and economic data can help portfolio managers to anticipate changes in interest rate levels. The resulting portfolios can produce a material incremental return relative to a passively structured portfolio.

FOOTNOTES

1. For the statistical analysis in this article, we used the BMDP 386 statistical software. For technical information on stepwise MDA, see the *BMDP Statistical Software Manual*, Vol. 1 (University of California Press).
2. The formula is $\sum_{n=45}^{70} (70!/[n!\,(70 - n)!])\, 0.5^{n} \times 0.5^{(70-n)}$.
3. R. McEnally and J. Jordan, "The Term Structure of Interest Rates," in F. Fabozzi, ed., *The Handbook of Fixed Income Securities*, 3rd ed. (Homewood, IL: Business One Irwin, 1991), and J. Hull, *Options, Futures, and Other Derivative Securities* (Englewood Cliffs, NJ: Prentice Hall, 1993).